Third European Set Theory Colloquium

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Mathematics and Philosophy

Study of consequences of ZFC and of non-consequences with a focus on combinatorial structures that are thought to be useful/ / interesting / relevant to study various universes of sets.

Trees, scales, squares, forcing orders, Ramsey theory, descriptive methods.

Other types of structures can come up and be successful for the solution of problems.

Search for models of ZFC, i.e., finitely/recursively axiomatizable relatively consistent extensions. Inner models or forcings extensions. Custom tailored models.

Analysis of a particular extension of ZFC by a known consistent statement or by a known notion of forcing.

Independence Results

ZFC result versus an independence result.

Run away and get back.

The Real Numbers and Subsets of $\mathbb{R}$

Convention: We consider only filters $\mathcal{F}$ over ω that contain all cofinite sets.

Definition

Let $\kappa \geq \aleph_1$ be a regular cardinal. A filter over ω called a P-filter if for any $\gamma < \aleph_1$ subset $\{A_\beta : \beta < \gamma\}$ of the filter, there is a $D \in \mathcal{F}$ with $(\forall \beta < \gamma)(D \subseteq^* A_\beta)$.

Question (Just, Mathias, Prikry, Simon)

Is there a non-meager P-filter? Known: The negation has consistency strength of a measurable.

Cardinal Invariants and Structure

Immense success without large cardinals.

Ten values in Cichon's diagramme.

Deep theory of properness.

Daily Work

The search for implications or forcings.

The more direct side of a forcing: Adding a tread to a scale, a diagonalisation of a filter.

Technical work on preservation.

Attraction to Rare Models

Work on ultrafilters in the countable support Cohen-avoiding scenario.

Most often outside the scope of a forcing axiom.

Diagonalisation Properties Without Dominating Sequences

Theorem (Bräuninger, M., to appear)

It is consistent relative to ZFC that there is a P-point with character $\aleph_1$ and a $P_{\aleph_2}$-point.

Theorem (M., to appear)

It is consistent relative to ZFC that there are exactly three near coherence classes of ultrafilters.

Preserved object of size $\aleph_1$, growth in length $\aleph_2$.

Many questions around it. Why only $\aleph_2$? What about finitely many classes? Some slowness also in the $\aleph_2$-direction. However, this is due to the construction.

Restrictions

Theorem (Raisonnier)

If the additivity of the Lebesgue null sets is $\geq \alpha_2$*, then there is a rapid ultrafilter.*

Thank you!