Notes from European Set Theory Colloquium

Thursday, May 11, 2023 7:37 AM

The past is the future

so many problems sit idle
because progress stops/slows
(self fulfilling)

keep pursuing partial results.

"Next big thing" is rare (subjective)

of course forcing, large cardinals

many suggestions over the years

certainly Scheeper's program

Arhangelskii's Cp theory

So, back to the past is the future

Some large cardinal related:

Normal Moore Space Conjecture

$+ \ c = \omega_2$

basically

normal + 1^{st} ctble $\Rightarrow$ collectionwise normal

1^{st} ctble $\aleph_1$-cwH $\Rightarrow$ cwH

Lindelof + points G_δ have card. $\leq 2^{\aleph_1}$

Def'n: Fréchet if $\overline{A} = A^{(1)}$

$A^{(1)} = A \cup \{x : \exists \langle a_n \rangle \xrightarrow{\subseteq A} x\}$

$$A^{(1)} = A \cup \{x : \exists \langle a_n \rangle \xrightarrow{=A} x\}$$

$$t(X) = \min\{\kappa : \overline{A} = \bigcup\{\overline{B} : B \in [A]^{\kappa}\}\}$$

How big can $t(X \times X)$ be if X is Fréchet

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opinion: adding compact makes questions more topological vs set-theoretic

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long standing external influence for Topology is

Does every compact subset of 2^{nd} dual have ...

G_δ-points, converging sequences, diverging sequences

defining Eberlein compact, Corson, Gulko etc

jump to

Efimov problem: Does every infinite compact space $\supset \omega+1$ or $\beta\omega$?

Efimov problem: ...

$\supseteq \omega+1$ or $\beta\omega$?

(related to $X^{*} \geq \aleph_1$)

partial? eg $\mathfrak{b} \neq \mathfrak{c}$

Moore-Mrowka + seq'l order

If compact X has $t(X) = \aleph_0$

does X have property $\overline{A} = (A^{(1)})^{(1)}$?

What if $\mathfrak{c} > \omega_2$? [? $\overline{A} = A^{(\omega_1)}$]

? mad f's

Simon: $(\text{compact Frechet})^2 \not\Rightarrow$ Frechet

because $\exists$ partitionable madf.

general investigation of $X \supset \omega_1$

small diagonal: $\Delta_X \subseteq X \times X$ (X cpt)

Does ω_1-inaccessible $\Delta \Rightarrow$ metrizable?

[<u>many</u> partial questions, few results]

Small Dowker: is there a normal X of cardinality $\aleph_1$ s.t. $X \times [0,1]$ is not normal?

Michael space: is there a Lindelof X s.t. $X \times$ irrationals is not Lindelof

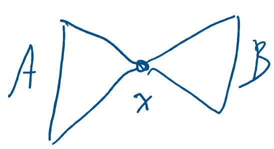
Of course: Can $\omega^* \approx \omega_1^*$

$$P(\omega)/fin \approx P(\omega_1)/fin$$

is $C(\omega^*)$ primary as a Banach space

$$\approx \ell_1/c_0$$

if $\omega^* = A \oplus_x B$, is one of $C(A)$, $C(B)$ $\sim C(\omega^*)$



$\vee$ $\sim C(\omega^{..})$

(bow-tie points may not exist)

points in ω^* $\subseteq$ topology (except q-points)

can $\omega^* \setminus \{p\}$ be normal

are there closed nwd $H \oplus_p K$

Does MA $\vdash$ $\mathcal{P}(\omega)/\text{fin}$ is c-universal

Def'n: seq'l cpt if every sequence has a converging subsequence

$x = \mathcal{U}\text{-}\lim\{x_n : n \in \omega\}$ if $x \in \overline{\{x_n : n \in U\}}$ for all $U \in \mathcal{U}$

Scarborough-Stone: is there a $\mathcal{U} \in \omega^*$ s.t. every seq'l compact space is $\mathcal{U}$-compact (every sequence has a $\mathcal{U}$-limit)

Frolik: ω^* is not <u>homogeneous</u>

Frolik: ω^* is not homogeneous

$orb_H(p) = X$ where H = group of autohomeomorphisms

Rudin's problem(s): Does every compact homogeneous X satisfy:

(1) contains a converging sequence

(2) has open chain condition $\leq c$

[van Douwen]

(3) will not map onto ω^*

[Kunen]

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author={Shakhmatov, Dmitri B.},
title={Compact spaces and their generalizations},
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title={Recent progress in general topology},
address={Prague},
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